**Practice Problems for Probabilistic Reasoning**

Some problems to help you prepare for Quiz 2 on Mon Oct 24.  
Please take help from friends and do post queries on moodle.  
Also any interesting lecture notes, videos, problems etc. you find.  
  
%%%% Probability Basics  
1. How many tosses of a fair coin are needed so that the probability of at  
   least one head is at least 0.99? H  
  
2. A family drives a camper across the country. Assume the number  
   of flat tires they will have during the trip is either 0 or 1 or 2 flat tires.  
   Also assume that the probability is 0.9 that they will have at most 1 flat  
   and the probability is 0.2 that they will have at least 1 flat.  
   Compute the probability that the family has  
       (a) no flat tires. (b) one flat tire. (c) two flat tires.  
         
3. A collection of 65 coins contains one with two heads; the remainder of the  
   coins are fair. If a coin, selected at random from the collection,  
   turns up heads 6 times in 6 tosses, what is the probability it is the two-headed coin?  
     
  
  
%% Conditional Probability and Bayesian Networks  
  
1.  After winning a race, an Olympic runner is tested for the presence of steroids.  
    The test comes up positive, and the athlete is accused of doping.  
    Suppose it is known that 5\% of all victorious Olympic runners do use performance-enhancing drugs.  
    For this particular test, the probability of a positive finding given that drugs are used is 95\%.  
    The probability of a false positive is 2\%. What is the (posterior) probability that the  
    athlete did in fact use steroids, given the positive outcome of the test?

2. Verify the example Bayes net calculations shown at  
   
            http://cs.nyu.edu/faculty/davise/ai/bayesnet.html  
  
3. Read       http://www.csse.monash.edu.au/bai/book/BAI\_Chapter2.pdf  
   and try Problem 3 at the end.  
  
  
%% Hidden Markov Models  
  
  
1. McDonalds and Burger King are the only restaurants in a small town. Both restaurants  
  compete with each other for customers. The probability a customer who ate at McDonalds  
  will eat there the next time is 0.8. The probability a customer who at at Burger King will  
  eat there the next time is only 0.4. If the town has 100 regular restaurant customers, after a  
  long time, how many customers will McDonalds have and how many will Burger King have.  
   
     (Check Wikipedia page on Markov chains and how to compute steady state probabilities)  
2. HMMs can be used to  decode  simple DNA sequences.  
   A DNA sequence is a series of symbols from A, C, G, T.  
   Assume there is one hidden variable S that controls the generation of DNA sequence.  
   S takes 2 possible states S1 , S2.  
   Assume the following transition probabilities for HMM M  
  
     P (S1 |S1 ) = 0.8, P (S2 |S1 ) = 0.2, P (S1 |S2 ) = 0.2, P (S2 |S2 ) = 0.8  
  
  and emission probabilities as following  
  
      P (A|S1 ) = 0.4, P (C|S1 ) = 0.1, P (G|S1 ) = 0.4, P (T |S1 ) = 0.1  
      P (A|S2 ) = 0.1, P (C|S2 ) = 0.4, P (G|S2 ) = 0.1, P (T |S2 ) = 0.4  
  
  and start probabilities as following  
  
     P (S1 ) = 0.5, P (S2 ) = 0.5  
  
   Assuming the observed sequence is x = CGT CAG, calculate  
    (a) The probability of observing this sequence given the HMM model.  
    (b) The probability of being in each state (S1, S2) after seeing each symbol.  
    (c) The most likely sequence of states that produced this observation.